eral correlation of heat transfer data for condensation of pure substances on horizontal pipes is given by the combination of Equations (1) and (3).

$$h_m \left(\frac{\mu_f^2}{k_f^3 \rho_f^2 g_c}\right)^{1/3} = 0.069 \ N_{Oh_f}^{-0.373} \ N_{Re_f}^{-1/3}$$

This relation contains the Nusselt analysis and the corrections for the effects of surface tension and pipe diameter which are important at the bottom of the pipe.

NOTATION

A = least squares curve fit constant
B = least squares curve fit constant
D = pipe diameter
g_c = gravitational constant
h_m = observed mean heat transfer coefficient
h_{Nu} = Nusselt's mean heat transfer coefficient
k = thermal conductivity
N_{Oh} = Ohnesorge number, $\mu/(\rho g_c D \sigma)^{1/2}$
N_{Re} = Reynolds number

Greek Letters

Γ' = mass rate of flow of condensate
μ = viscosity
ρ = density
σ = surface tension

Superscript

f = physical property evaluated at film temperature

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Distillation in Packed Columns: The Relationship Between HTU and Packed Height

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HTU (or HETP) in packed distillation columns has been found to vary with packed height. The explanations that have been given are improper evaluation of end effects (*1*); phase channeling, that is, a small portion of the liquid flows down through the bed without any appreciable mass transfer between the phases (*2*); or the variation of mass transfer area with packed bed height (3).

It is the purpose of this communication to show quantitatively that longitudinal mixing (back mixing) of the phases is an important factor that determines the relation between HTU and packed height in the case where end effects have been evaluated properly and the liquid is distributed evenly ensuring uniform mass transfer area throughout the bed.

LONGITUDINAL MIXING MODEL

The model used to derive the relationship between HTU and packed height is the model developed by Miyauchi and Vermeulen (*4*) for two-phase flow operations. In this model the longitudinal mixing in each phase is characterized by a single parameter: the eddy dispersion coefficient.

The basic differential equations can be derived from steady state material balance for a differential element of the bed height *dz*.

For the gas phase

$$D_y \frac{d^2y}{dz^2} - G_M \frac{dy}{dz} - K_g a \, (y^* - y) = 0 \qquad (1a)$$

and for the liquid phase

$$D_x \frac{d^2x}{dz^2} + L_m \frac{dx}{dz} + K_g a \, (y^* - y) = 0 \qquad (1b)$$

$K_g a$ is the point mass transfer coefficient and its value is assumed to be constant throughout the bed.

The boundary conditions are

$$z = 0 \ : \ G_M y_B = G_M \ y - D_y \frac{dy}{dz} \qquad (2a)$$

$$\frac{dx}{dz} = 0 \qquad (2b)$$

$$z = 1 \ : \ L_M \, x_D = L_M \ x + D_x \frac{dx}{dz} \qquad (2c)$$

$$\frac{dy}{dz} = 0 \qquad (2d)$$

In deriving the boundary conditions, it has been assumed that the mixing in the bed is much higher than that outside the bed, and mass transfer at the ends of the bed has been neglected.

Hartland and Mecklenburgh (3) recently have presented analytical solutions of the above equations. Their results were derived for extraction, but they are applicable to the present case. The concentration profile of the vapor along the bed is given by Equation XII of their paper. The vapor composition leaving the bed is obtained by putting $z = 1$ into Equation XII. Knowing the vapor and liquid compositions entering and leaving the bed, we can therefore calculate $K_g a$ and hence HTU_{OG}, provided the flow rate and the degree of longitudinal mixing of each phase are also known.

In practice, however, HTU_{OG} is calculated from the end compositions by assuming plug flow in both phases, that is, from Equation XXXX of Hartland paper with $z = 1$. Let us denote HTU_{OG} calculated by the later method by HTU_{OGP}. The relationship between HTU_{OGP} and packed height H is obtained by equating Equations XII and XXXX with $z = 1$.

$$HTU_{OGP} = \frac{H(1-\lambda)}{\ln \dfrac{1-(1-\lambda)(1-\psi)}{\psi}} \qquad (3)$$

where

$\lambda = m\ G_M/L_M$

ψ = right-hand side of Equation XII.

EXAMPLE

Yoshida et al. (6) reported measurements of HTU_{OGP} in the distillation of benzene toluene mixtures at total reflux in a 15-cm. diameter column packed with 15-mm. Raschig rings at three different packed heights. These data are chosen because correlations of eddy dispersion data are available for ½-in. Raschig rings (7, 8).

The true HTU calculated at vapor flow rate of 5.0 lb.-moles/(hr.)(sq.ft.) and packed height of 3.4 ft. (largest height), where end effects are minimum, is 0.36 ft. The following numerical values are used:

$$N_{Pe} \text{ of the vapor phase} = \frac{G_M d_P}{D_y} = 0.57$$

$$N_{Pe} \text{ of the liquid phase} = \frac{L_M d_P}{D_x} = 0.070$$

$$y^* = 0.58\,x + 0.44 \text{ (for } x = 0.4 \text{ to } 0.9)$$

The liquid phase eddy dispersion coefficient is extrapolated from the correlation of Sater and Levenspiel (7) and the vapor phase eddy dispersion coefficient from the correlation of De Maria and White (8). These data were for air water operation in a 4-in. column packed with ½-in. Raschig rings.

If Sater's values of N_{Pe} for the gas and liquid phases are used, the calculated true value of HTU_{OG} is negative, which indicates that his correlations predict too high values of the eddy dispersion coefficient. However, there is considerable scatter in his data and if the upper limit value for the liquid phase is used, together with the result of De Maria for the gas phase, then a positive and reasonable value of HTU_{OG} is obtained.

TABLE 1. EFFECT OF PACKED HEIGHT ON HTU_{OGP}

	HTU_{OGP}, ft.	
Packaged height, ft.	Measured	Predicted
0.13	0.44	0.43
1.77	0.74	0.74
3.40	0.82	0.81

Using the calculated true HTU_{OG}, we can calculate the HTU_{OGP} at the other two packed heights. The results are shown in Table 1.

It is seen that a nearly twofold variation of HTU_{OGP} with packed height is accounted for by the longitudinal mixing in the bed.

CONCLUSIONS

1. Longitudinal mixing in the liquid and vapor phases is an important factor affecting the relationship between HTU_{OGP} and packed height.
2. HTU_{OGP} measured in experimental columns with small heights cannot be used directly in the design of industrial scale large height packed columns. The true HTU_{OG} has to be calculated first and then HTU_{OGP} at the larger height is calculated by Equation (3).
3. Presently available correlations of HTU_{OGP} are based on experimental data from columns with different heights. Improved correlations could be obtained if the true HTU_{OG} is correlated instead of HTU_{OGP}.

Experimental work is in progress to measure the performance of packed columns with variable height and to measure eddy dispersion coefficients to extend the available correlations.

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NOTATION

D = eddy dispersion coefficient, lb.-moles/(hr.)(ft.)
G_M = molal gas flow rate, lb.-moles/(hr.)(sq.ft.)
H = packed height, ft.
$K_g a$ = mass transfer coefficient based on the vapor phase, lb.-moles/(hr.)(cu.ft.)
L_M = molal liquid rate, lb.-moles/(hr.)(sq.ft.)
m = slope of equilibrium line
x = mole fraction of the more volatile component in the liquid phase
y = mole fraction of the more volatile component in the vapor phase
z = fractional distance along bed

Subscripts

x = liquid phase
y = vapor phase

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